New approaches to deghosting towed-streamer and ocean-bottom pressure measurements

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Summary

A variant of the extinction theorem is combined with a recent wavefield prediction technique (Tan, 1999, Weglein et al., 2000) to produce a direct method for deghosting towed streamer data through an integral of the pressure measurements along the cable. Only those pressure measurements are required. Neither finite difference, nor Taylor Series type approximations, nor dual streamer measurements are required. Single sensor towed streamer acquisition is a practical prerequisite.

We also present a separate method for deghosting oceanbottom pressure measurements (deeper than 10 meters) that requires the source signature in water but completely avoids the need to measure the vertical particle velocity.

Background

Deghosting and wavelet estimation have risen in importance as prerequisites for free surface and internal multiple removal and for resolution and delineation of imaged-inverted primaries. The industry trend to deep water exploration/production and the increased use of ocean bottom measurements have added significantly to the interest in developing more robust and effective procedures. For example, in ocean bottom measurements the ghost notch is well within the seismic bandwidth and represents a serious obstacle to reaching processing and E and P objectives.

In principle, separating up and down waves from the source or arriving at the receiver can be achieved by either one measurement at two depths or two measurements at one depth (see, e.g., Schneider et al., 1964, Barr and Sanders, 1989, Fokkema and van den Berg, 1993, Amundsen, 1993, Dragoset and Barr, 1994). Sensitivity to errors in depth estimation is a serious problem for the former approach and hydrophone-geophone coupling, instrument response differences, and noise sensitivity can have deleterious effects on the latter (Ball and Corrigan, 1996).

This paper is written as a response to the challenges described for both towed streamer and ocean bottom pressure measurements. It seeks to benefit from the strength of the dual sensor summation technique while avoiding the pitfalls. For example, in the case of the oceanbottom pressure data, it directly predicts rather than measures the vertical derivative of pressure from the pressure itself and the source signature. The source signature (wavelet) can be determined directly from towed streamer data or a near-field measurement and far-field extrapolation (see, e.g., Ziolkowski, 1980) and we anticipate that it could be more robust than the measurement of vertical particle velocity transformed into vertical derivative of pressure.

Wavelet estimation, wavefield prediction and the extinction theorem for deghosting marine data

Let the actual pressure wavefield satisfy

$$\nabla^2 P + \frac{\omega^2}{c_0^2} (1 - \alpha(\mathbf{r})) P = A(\omega) \delta(\mathbf{r} - \mathbf{r}_s)$$
(1)
where
$$1 \qquad 1$$

$$\frac{1}{c^{2}(r)} = \frac{1}{c_{0}^{2}} (1 - \alpha(r))$$

 $c(\mathbf{r})$ = velocity configuration, and c_0 = acoustic wavespeed in water. We will use Green's theorem with *P* and different Green's functions, *g*:

$$\int_{V} \left(P \nabla'^{2} g - g \nabla'^{2} P \right) d\mathbf{r}' = \oint_{S} \left(P \frac{\partial g}{\partial n'} - g \frac{\partial P}{\partial n'} \right) dS'^{(2)}.$$

In Weglein and Secrest (1990), the reference medium is chosen as a half-space of water bounded by a free surface (FS) at the air-water interface. Choose $g = G_0^D$ to be the causal Dirichlet Green's function that vanishes at the free surface, and substitute Eq. (1) into (2): $A(\omega)G_0^D(\mathbf{r},\mathbf{r}_{-\omega}\omega)$ (2)

$$A(\omega)G_{\theta}^{D}(\mathbf{r},\mathbf{r}_{s},\omega)$$
(3)
= $\int_{M} (P(\mathbf{r}',\mathbf{r}_{s},\omega) \frac{\partial G_{\theta}^{D}(\mathbf{r},\mathbf{r}',\omega)}{\partial z'} - G_{\theta}^{D}(\mathbf{r},\mathbf{r}',\omega) \frac{\partial P(\mathbf{r}',\mathbf{r}_{s},\omega)}{\partial z'}) dS'$

where *P* and G_0^D vanish on the free surface and *r* is below the measurement surface, M. In practical applications, Eq. (3) provides a plethora of estimates of $A(\omega)$ (each *r* below M provides another estimate) that result in a robust prediction of the amplitude and phase of the source wavelet (see, e.g., De Lima et al., 1990). It requires *P* and dP/dzalong the cable and single sensor measurements. H. Tan (1992 and 1999) and A. Osen et al (1998) extended this idea by choosing a Green's function G_0^{DD} in (2) that vanishes at both the free surface and the measurement surface M (Fig. 1). H. Tan (1999) recognized that the combination of G_0^{DD} and a towed streamer at ~6 m depth, and for f < 125 Hz, Eq. (3) provides an accurate prediction of the total field, P, above the cable in terms of only P on the cable and without knowing the source function $A(\omega)$:

$$P(\mathbf{r},\mathbf{r}_{\mathbf{s}},\boldsymbol{\omega}) \cong \int_{M} dS' P(\mathbf{r}',\mathbf{r}_{\mathbf{s}},\boldsymbol{\omega}) \frac{\partial G_{\theta}^{DD}(\mathbf{r},\mathbf{r}',\boldsymbol{\omega})}{dz'} \quad (4),$$

and

$$\frac{\partial P(\boldsymbol{r},\boldsymbol{r}_{s},\boldsymbol{\omega})}{\partial z} \cong \int_{M} dS' P(\boldsymbol{r}',\boldsymbol{r}_{s},\boldsymbol{\omega}) \frac{\partial^{2} G_{0}^{DD}(\boldsymbol{r},\boldsymbol{r}',\boldsymbol{\omega})}{\partial z \partial z'}$$
(4')

where r = (x,z) is a point between the cable and the free surface.

Extinction Theorem for towed-streamer deghosting

Consider the same marine experiment in Figure 1, but now choose the reference as a whole space of water, with $g=G_0^+$, the causal whole space Green's function and three sources: one active, representing the air-guns, $\rho_s=A(\omega) \, \delta(r-r_s)$, and two passive, $\rho_a=k^2 \alpha_a(r) H(-z)$, and $\rho_e=k^2 \alpha_e(r) H(z-z_b)$, turning water into air and water into earth, respectively. The free surface and the water bottom are at z=0 and below z_b , respectively. With this choice of

$$g = G_0^+(\mathbf{r}, \mathbf{r}_s, \omega) = -\frac{1}{4\pi} \frac{e^{i\mathbf{k}|\mathbf{R}|}}{\mathbf{R}},$$

where $\mathbf{R} = |\mathbf{r} - \mathbf{r}_s|,$
 $\nabla^2 P + k^2 P = k^2 \alpha_e P + k^2 \alpha_a P + A(\omega) \delta(\mathbf{r} - \mathbf{r}_s)$

$$= \rho_e + \rho_a + \rho_s,$$

and $k = \omega/c_0$. Choosing **r** to be any point in the volume above *M* in Figure (2), Eq. (2) becomes

$$\int \rho_{e} G_{0}^{+} d\mathbf{x}' d\mathbf{z}' = \int_{M} \left(P \frac{\partial G_{0}^{+}}{\partial \mathbf{z}'} - G_{0}^{+} \frac{\partial P}{\partial \mathbf{z}'} \right) d\mathbf{S}' \qquad (5)$$

Since



Figure 1: The extinction theorem can be used to calculate the source wavelet from marine streamer pressure measurements.



$$\int \rho_e G_0^{\dagger} dx' dz$$

= $\int_{z > z_b} G_0^{\dagger}(x, z, x', z', \omega) k^2 \alpha_e(x', z') P(x', z', x_s, z_s, \omega) dr$

represents propagation straight from the earth (subsurface), ρ_{e} , to the receivers, then it follows that Eq. (5) is receiverdeghosted data.

For towed streamer data we can use Eq. (4') to compute $\partial P/\partial z'$ directly from *P* on the cable. The surface integral Eq. (5) removes the direct wave, the direct wave ghost, all ghosts on the receiver side and all source ghosts that have a receiver ghost. A second application of the extinction theorem over the source coordinate removes the isolated source ghosts. The completely deghosted reflected wavefield, P_d^{T} , for towed streamer data is given directly by an integral of the total wavefield, *P*.

$$P_{d}^{r}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{x}',\boldsymbol{z}',\boldsymbol{\omega}) = \int d\boldsymbol{x}_{s} \lim_{\boldsymbol{\varepsilon} \to 0^{+}} \int W_{2}(\boldsymbol{x}',\boldsymbol{z}',\boldsymbol{x}_{s},\boldsymbol{x}'_{s},\boldsymbol{z}_{s},\boldsymbol{\omega}) d\boldsymbol{x}'_{s}$$

$$\times \int d\boldsymbol{x}_{g} \lim_{\boldsymbol{\varepsilon} \to 0^{+}} \int W_{2}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{x}_{g},\boldsymbol{x}'_{g},\boldsymbol{z}_{c},\boldsymbol{\omega}) P(\boldsymbol{x}'_{g},\boldsymbol{z}_{c},\boldsymbol{x}'_{s},\boldsymbol{z}_{s},\boldsymbol{\omega}) d\boldsymbol{x}'_{g}$$
(6)

where z_c and z_s are the depths of the cable and the source, respectively, and

$$W_{2}(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{x}_{g}, \boldsymbol{x}'_{g}, \boldsymbol{z}_{c}, \boldsymbol{\omega}) = \delta(\boldsymbol{x}'_{g} - \boldsymbol{x}_{g}) \left[\frac{\partial \boldsymbol{G}_{0}^{+}(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{x}'_{g}, \boldsymbol{z}', \boldsymbol{\omega})}{\partial \boldsymbol{z}'} \right]_{\boldsymbol{z}' = \boldsymbol{z}_{c}} - \boldsymbol{G}_{0}^{+}(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{x}_{g}, \boldsymbol{z}_{c}, \boldsymbol{\omega}) \left[\frac{\partial^{2} \boldsymbol{G}_{0}^{DD}(\boldsymbol{x}_{g}, \boldsymbol{z}', \boldsymbol{x}'_{g}, \boldsymbol{z}'', \boldsymbol{\omega})}{\partial \boldsymbol{z}' \partial \boldsymbol{z}''} \right]_{\boldsymbol{z}' = \boldsymbol{z}_{c}}$$

where (x,z) and (x',z') represent the coordinates of the predicted receiver and source respectively. Eq. (6) actually computes much more than source and receiver deghosted

Deghosting streamer and OBC data

data along the cable - it in fact predicts what source and receiver deghosted data would be at all receiver positions above the cable and for all source points above the actual source depth. The use of this additional predicted information will be the subject of future work.

Ocean bottom deghosting of pressure measurements

We cannot predict $\partial P/\partial z$ for cable depths greater than 10 m using Eq. (4'), i.e. directly from pressure *P* along the cable, and hence Eq. (6) is not available. Using a combination of hydrophones and geophones often suffers from issues related to geophone coupling, different instrument responses and noise sensitivities. For ocean bottom pressure measurements we propose using the triangle relationship between $A(\omega)$, *P*, and $\partial P/\partial z$ given by Eq. (3) to solve for $\partial P/\partial z$ from measured values of *P* and measured/predicted knowledge of $A(\omega)$.

 $A(\omega)$ could be directly determined from towed-streamer pressure measurements by combining Eqs. (3) and (4'):

$$A(\omega) = \frac{\int dx_g \lim_{\varepsilon \to 0^+} \int dx'_g W_1(x, z, x'_g, x_g, z_c, \omega) P(x'_g, z_c, x_s, z_s, \omega)}{G_0^D(x, z, x_s, z_s, \omega)}$$

where

$$W_{1}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{x}'_{g},\boldsymbol{x}_{g},\boldsymbol{z}_{c},\boldsymbol{\omega}) = \delta(\boldsymbol{x}'_{g} - \boldsymbol{x}_{g}) \left[\frac{\partial \boldsymbol{G}_{0}^{D}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{x}'_{g},\boldsymbol{z}',\boldsymbol{\omega})}{\partial \boldsymbol{z}'} \right]_{\boldsymbol{z}'=\boldsymbol{z}_{c}}$$
$$-\boldsymbol{G}_{0}^{D}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{x}_{g},\boldsymbol{z}_{c},\boldsymbol{\omega}) \left[\frac{\partial^{2} \boldsymbol{G}_{0}^{DD}(\boldsymbol{x}_{g},\boldsymbol{z}',\boldsymbol{x}'_{g},\boldsymbol{z}'',\boldsymbol{\omega})}{\partial \boldsymbol{z}'\partial \boldsymbol{z}''} \right]_{\boldsymbol{z}'=\boldsymbol{z}_{c}}$$

or from direct near-field measurements and far-field extrapolation. Given $A(\omega)$ and $P(x,z_{cr}x_{sr}z_{sr}\omega)$ we compute $\partial P/\partial z$ after Fourier transforming $\int e^{-ip_1 x}$ both sides of Eq. (3) over x (This is a 2D result – the 3D generalization is straightforward):

$$\frac{q}{\sin q z_{t}} \left[A(\omega) e^{-i\rho_{x_{s}}} \left(-\frac{\sin q z_{s}}{q} \right) + 2\cos q z_{t} P(p_{1}, z_{c}, x_{s}, z_{s}, \omega) \right]$$
$$= \left[\frac{\partial P(p_{1}, z', x_{s}, z_{s}, \omega)}{\partial z'} \right]_{z' = z_{c}}$$

where $q^2 = (\omega^2/c_0^2 - p_1^2)$. We then use *P* and $\partial P/\partial z'$ in Eq. (5) to receiver-deghost the ocean-bottom pressure measurements. Again, a second application of Eq. (5) over

sources of the receiver-deghosted data results in reflection data with all ghosts removed.

Conclusions

We have presented two new and distinct approaches for deghosting towed-streamer pressure measurements and ocean-bottom hydrophones, respectively. In the former case the algorithm requires only the total pressure field measurements on the cable and directly outputs totally deghosted reflection data. In the latter case the procedure requires either a prediction or measurement of the source signature plus the total pressure field measurements on the cable and outputs deghosted data.

When the wavelet is determined from an integral over the towed streamer pressure data, Eq. (7), it would contain an instrument response that could match the instrument response in the deep hydrophone pressure measurement, and, hence, lead to a calculation of deghosted ocean bottom reflection data that accommodates the instrument response.

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